THE WEST AFRICAN EXAMINATIONS COUNCIL

West African Senior School Certificate Examination

MATHEMATICS (CORE) 2

November 2010

[100 marks]

2½ hours

Write your name and index number in ink in the spaces provided above.

Answer ten questions in all. All the questions in Part I and five questions from Part II.

In each question, all necessary details of working, including rough work, must be shown with the answer.

Give answers as accurately as data and tables allow.

Graph papers are provided for your use in the examination.

The use of non-programmable, silent and cordless calculator is allowed.
1. (a) Simplify \(37\frac{1}{2} + \frac{5}{9}\) of \((4\frac{1}{7} + 1\frac{1}{5}) - 80\frac{1}{3}\).

(b) The \(n\)th term of a sequence is \(5 + \frac{2}{3^n - 1}\) for \(n \geq 1\). What is the sum of the fourth and fifth terms? Leave your answers in the form \(\frac{x}{y}\) where \(x\) and \(y\) are integers.

2. (a) Factorize \(x^2 + 4x + 3 + mx + 3m\).

(b) A T-shirt costs 5 times as much as a singlet. For GH₵800, a trader can buy 32 more singlets than T-shirts. How much does a T-shirt cost?

3. The diagram shows a field \(ABCD\) in the form of a trapezium.
If \(|AD| = |BC| = 600\ m, \angle ADC = \angle BCD = 120\degree\) and \(|DC| = 450\ m,\)
(a) find the perimeter of the field;
(b) calculate, correct to three significant figures, the area of the field.

4. In the diagram, \(|OR| = 5\ m, \angle ORP = 45\degree, \angle OQP = 60\degree\) and \(\angle OPR = 90\degree\).
Find the distance \(QP\), leaving your answer in surd form.

(b) \(X\) and \(Y\) are two cylindrical tanks with base radii \(2r\ cm\) and \(r\ cm\) respectively. If the water level in \(Y\) is 10 cm, what level will the same quantity of water be in \(X?\) (Use \(\pi = \frac{22}{7}\))

5. (a) In a class, the probability that a student passes a test is \(\frac{2}{5}\). What is the probability that if 2 students are chosen at random from the class, one would pass and the other would fail?

(b) In the diagram, \(O\) is the centre of the circle. \(AK\) is a straight line and \(TK\) is a tangent. If \(\angle CTK = 30\degree\), calculate \(\angle TKC\).

PART II
[60 marks]

Answer five questions only from this part. All questions carry equal marks.

6. (a) In a class of 31 students, 16 play football, 12 play table-tennis and 5 play both games. Find the number of students who play
(i) at least one of the games;
(ii) none of the games.

(b) Two commodities \(A\) and \(B\) cost D70 and D80 per kg respectively. If 34.5 kg of \(A\) is mixed with 26 kg of \(B\) and the mixture is sold at D85 per kg, calculate the percentage profit.
7. (a) Copy and complete the following table of values for the relation 
\[ y = (x - 4)(x + 2) \] for \(-3 \leq x \leq 5\).

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Using scales of 2 cm to 1 unit on the x-axis and 2 cm to 2 units on the y-axis, draw the graph of \( y = (x - 4)(x + 2) \) for \(-3 \leq x \leq 5\).

(c) Using the graph, find the:
   (i) values of \( x \) for which \( y \) is decreasing;
   (ii) gradient of the curve at \( x = 0 \).

8. (a) Using a ruler and a pair of compasses only, construct:
   (i) triangle \( PQR \) such that \( |PQ| = 8.5 \text{ cm}, \angle QPR = 60^\circ \) and \( |PR| = 7.5 \text{ cm} \);
   (ii) the locus \( l_1 \) of points equidistant from \( P \) and \( R \);
   (iii) the locus \( l_2 \) of points equidistant from \( Q \) and \( R \);
   (iv) locate the point of intersection \( J \) of the loci \( l_1 \) and \( l_2 \).

(b) (i) Construct a circle passing through the three vertices of the triangle \( PQR \).
   (ii) Find the radius of the circle.
   (iii) Measure \( |QR| \).

9. (a) Simplify \( \sqrt{\frac{x^2 + y^2}{xy}} \), where \( x > 0 \) and \( y > 0 \).

(b) A man 1.7 m tall observes the angle of elevation of the tip of a tower to be 35°. He moves 50 m away from the tower and now observes the angle of elevation to be 28°. How far above the ground is the tip of the tower to three significant figures?

10. The table shows the distribution of marks scored by 50 students in a test.

<table>
<thead>
<tr>
<th>Marks (%)</th>
<th>1 - 10</th>
<th>11 - 20</th>
<th>21 - 30</th>
<th>31 - 40</th>
<th>41 - 50</th>
<th>51 - 60</th>
<th>61 - 70</th>
<th>71 - 80</th>
<th>81 - 90</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>8</td>
<td>12</td>
<td>10</td>
<td>5</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

(a) Construct a cumulative frequency table for the distribution.

(b) Draw a cumulative frequency curve for the distribution.

(c) Use the curve to estimate the:
   (i) interquartile range;
   (ii) percentage of students who scored more than 66%.

11. The base of a right pyramid with vertex \( O \) is a square \( ABCD \) of side 13 cm. Each slant edge is 12 cm long. Calculate, correct to two significant figures, the:
   (a) vertical height \( |OE| \) of the pyramid;
   (b) volume of the pyramid.

12. (a) The operation \( \bullet \) is defined on the set \( \{2, 4, 6\} \) by \( m \bullet n \) = the unit digit in the product \( mn \).
   (i) Copy and complete the table.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(ii) Use the table to solve the following equations:

(a) \( x \cdot 4 = 8 \);
(b) \( e \cdot e = e \);
(c) \( (4 \cdot f) \cdot 4 = f \).

(b) The functions \( f \) and \( g \) are defined as

\( f : x \rightarrow 2 - x^2 \) and
\( g : x \rightarrow \frac{1}{x-1} \).

Evaluate

(i) \( g(-\frac{1}{4}) \);
(ii) \( \frac{f(2)}{g(3)} \).

13. (a) A triangle has vertices \( A(1, 1) \), \( B(2, 4) \) and \( C(5, 8) \).

(i) If the triangle is translated by the vector \( \left( \begin{array}{c} 1 \\ -1 \end{array} \right) \) to \( A'B'C' \), where \( A \rightarrow A' \), \( B \rightarrow B' \) and \( C \rightarrow C' \), calculate the coordinates of \( A', B' \) and \( C' \).

(ii) The triangle \( ABC \) undergoes a transformation involving rotation in an anticlockwise direction through \( 90^\circ \) about the origin followed by a translation. If the final position is \( A''(2, -1), B''(-1, 0) \) and \( C''(-5, 3) \), determine the translation vector.

(b) In triangle \( PQR \), \( \overrightarrow{PQ} = \left( \begin{array}{c} 3 \\ 2 \end{array} \right) \) and \( \overrightarrow{RQ} = \left( \begin{array}{c} -6 \\ -4 \end{array} \right) \), find \( \overrightarrow{PR} \).

(c) Find the equation of the line which is perpendicular to the line \( y = 2x - 1 \) and passes through the point \( (2, 5) \).

QUESTIONS 14 AND 15 ARE FOR CANDIDATES IN NIGERIA, SIERRA LEONE AND THE GAMBIA ONLY.

14. \( P \) (lat \( 40^\circ \) N, long \( 18^\circ \) W) and \( Q \) (lat \( 40^\circ \) N, long \( 78^\circ \) W) are two cities on the surface of the earth. Calculate the:
(a) radius of the parallel of latitude on which \( P \) and \( Q \) lie, correct to the nearest \( 10 \) km;
(b) length of the minor arc \( PQ \), correct to the nearest \( 100 \) km;
(c) vertical distance between the centre of the earth and the centre of the small circle on which \( P \) and \( Q \) lie, correct to the nearest km.

[Take \( \pi = \frac{22}{7} \) and radius of the earth = 6400 km]

15. (a) The second, fourth and sixth terms of an Arithmetic Progression (AP) are \( x - 1 \), \( x + 1 \) and 7 respectively. Find the
(i) common difference;
(ii) first term;
(iii) value of \( x \).

(b) A spherical bowl of radius \( r \) cm is one-quarter full when 6 litres of water is poured into it. Calculate, correct to three significant figures, its diameter. [Take \( \pi = \frac{22}{7} \)].

END OF PAPER
Visit www.larnedu.com now for more!